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A consistent description of $\rho^0 \to \pi\pi\gamma$ decays including $\sigma(500)$ meson effects

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Abstract: A consistent description of $\sigma(500)$ meson effects in $\rho^0 \to \pi^0\pi^0\gamma$ and $\pi^+\pi^-\gamma$ decays is proposed in terms of reasonably simple amplitudes which reproduce the expected chiral-loop behaviour for large m_σ values. For the neutral case, in addition to the well known ω exchange, there is an important contribution from the $\sigma(500)$ meson that is in agreement with recent experimental data. For the charged case, where the dominant contribution comes from bremsstrahlung, the effects of the $\sigma(500)$ meson are relevant only at high values of the photon energy and compatible with present data. A combined analysis of both processes with moderately improved experimental information should contribute decisively to clarify the status of this controversial $\sigma(500)$ meson.

Keywords: Phenomenological Models, Chiral Lagrangians.

Contents

1. Introduction

If there is a meson resonance whose existence or not is still an open question in spite of many dedicated discussions, this is the $\sigma(500)$ meson. Although the current PDG edition [1] classifies this scalar state — the σ or $f_0(400\text{–}1200)$ — among the stablished resonances, this has not been the case for most of a controversial period starting some 30 years ago. Data on $\pi\pi$ scattering at low energies, whose isoscalar s-wave channel should reflect the $\sigma(500)$ effects and allow for the extraction of the $\sigma(500)$ properties, have resisted unambiguous analyses. Only recently, a growing number of authors claim for the existence of such a $\pi\pi$ resonant state with a mass around some 500 MeV and a similar width (for two recent reviews, see refs. [2] and [3]). But the controversy on the existence of the $\sigma(500)$, as well as on its nature and properties, is still open. The purpose of our note is to illustrate that a combined analysis of the radiative $\rho^0 \to \pi^0\pi^0\gamma$ and $\rho^0 \to \pi^+\pi^-\gamma$ decays should considerably contribute to clarify the issue.

The contribution of the $\sigma(500)$ meson to the amplitudes of these two radiative processes is exactly the same under isospin invariance, $\mathcal{A}(s)_\sigma \equiv \mathcal{A}(\rho \to \pi^0\pi^0\gamma)_\sigma = \mathcal{A}(\rho \to \pi^+\pi^-\gamma)_\sigma$. It should be the dominant one in the s-channel ($s \equiv m^2_{\pi\pi} \leq m^2_\rho$) where two-pion resonance formation with $J^{PC} = 0^{++}, 2^{++}\ldots$ can occur. Indeed, while the $\sigma(500)$ has a mass below m_ρ and strongly couples to pion pairs, other resonant exchanges have too large a mass (like the $f_2(1270)$) or almost decouple from pions (like the $f_0(980)$). This common σ amplitude — the *signal* amplitude, $\mathcal{A}(s)_\sigma$ — will interfere with other contributions — *background* amplitudes — accounting for other exchanges. The latter can be reliably computed for both the neutral and charged decays and turn out to be markedly different. Thanks to this, the combined study of both decays and comparison with their data considerably constraints the common signal amplitude, $\mathcal{A}(s)_\sigma$, and should allow for the extraction of the $\sigma(500)$ meson properties. With the available data on $\rho^0 \to \pi^0\pi^0\gamma$ [4, 5] and $\rho^0 \to \pi^+\pi^-\gamma$ [6, 7] one already can infer that a low-mass $\sigma(500)$ resonance is most likely required. More accurate data coming from the Frascati ϕ-factory DAΦNE [8] could confirm this conclusion and extract the relevant $\sigma(500)$ meson properties.

2. The common, $\sigma(500)$-dominated amplitude

The suitable tool to study e^+e^- annihilation into $\pi^0\pi^0\gamma$ or $\pi^+\pi^-\gamma$ well below the ρ resonance pole is Chiral Perturbation Theory (χPT) [9]. At the one-loop level, this would imply the extension of the analysis on $\gamma\gamma \to \pi^0\pi^0$ and $\gamma\gamma \to \pi^+\pi^-$ for real photons performed in ref. [10] to the case where one photon is off mass-shell $(q^*)^2 \neq 0$. But if the ρ resonance pole is approached, $(q^*)^2 \simeq m_\rho^2$, χPT no longer applies and one needs to enlarge this theory to include resonances. There is some consensus in that vector and axial-vector mesons have to be incorporated in such a way that the old and successful ideas of Vector Meson Dominance (VMD) are fulfilled [11], but the situation concerning the inclusion of scalars is notoriously more ambiguous [11, 12].

Extensions of χPT including vector mesons which are suitable for the analysis of $\rho^0 \to \pi\pi\gamma$ radiative decays have been presented elsewhere [13, 14]. They are particularly simple when specified to the neutral decay mode $\rho^0 \to \pi^0\pi^0\gamma$. In this case one has to compute the same set of one-loop diagrams contributing to $\gamma\gamma \to \pi^0\pi^0$ shown in ref. [10], the only difference being the substitution of one photon with $(q^*)^2 \neq 0$ by a massive ρ meson according to VMD. Restricting to the contribution from charged-pion loops (charged-kaon loops contribute negligibly [10, 14]) one obtains the following finite amplitude for $\rho(q^*, \epsilon^*) \to \pi^0(p)\pi^0(p')\gamma(q, \epsilon)$:

$$\mathcal{A}(\rho \to \pi^0\pi^0\gamma)_\chi = \frac{-eg}{\sqrt{2}\pi^2 m_{\pi^+}^2}\{a\}\, L(m_{\pi^0\pi^0}^2) \times \mathcal{A}(\pi^+\pi^- \to \pi^0\pi^0)_\chi\,, \tag{2.1}$$

where $\{a\} \equiv (\epsilon^* \cdot \epsilon)\,(q^* \cdot q) - (\epsilon^* \cdot q)\,(\epsilon \cdot q^*)$, $m_{\pi^0\pi^0}^2 \equiv s \equiv (p+p')^2 = (q^* - q)^2$ is the invariant mass of the final dipion system and $L(m_{\pi^0\pi^0}^2)$ is the loop integral function defined in refs. [14]–[17]. The coupling constant g comes from the strong amplitude $\mathcal{A}(\rho \to \pi^+\pi^-) = -\sqrt{2}g\, \epsilon^* \cdot (p_+ - p_-)$ with $|g| = 4.24$ to agree with $\Gamma(\rho \to \pi^+\pi^-)_{\rm exp} = 149.2\,\mathrm{MeV}$ [1]. The final factor in eq. (2.1) is

$$\mathcal{A}(\pi^+\pi^- \to \pi^0\pi^0)_\chi = \frac{s - m_\pi^2}{f_\pi^2}\,. \tag{2.2}$$

Although it is the part of the amplitude which is potentially sensitive to the effects of σ resonance formation, this is not contemplated in our chiral-loop evaluation at lowest order. By itself this $\mathcal{A}(\pi^+\pi^- \to \pi^0\pi^0)_\chi$ amplitude in eq. (2.1) — devoid of σ formation effects — leads to

$$\Gamma(\rho \to \pi^0\pi^0\gamma)_\chi = 1.55\,\mathrm{keV}\,, \tag{2.3}$$

for $f_\pi = 92.4\,\mathrm{MeV}$. It is worth mentioning that the amplitude (2.1) is calculated by means of the $\mathcal{O}(p^2)$ χPT lagrangian, $\mathcal{L}_2$, enlarged to include external vector meson fields through the covariant derivative. In this sense, the $\pi^+\pi^- \to \pi^0\pi^0$ amplitude in eq. (2.2) is correct only at lowest order in the chiral expansion. A more refined two loop analysis including terms of the $\mathcal{O}(p^4)$ χPT lagrangian, $\mathcal{L}_4$, would make the amplitude (2.2) no longer proportional to $(s - m_\pi^2)$ but corrected by chiral loop and counterterm contributions [18, 19]. Some of these counterterms are known to contain the effects of scalar resonance exchange [11]. If one is only interested in such effects, as in our present case, it has been shown very recently that a direct comparison of $\pi\pi$ scattering in the Linear Sigma

Model (LσM) and χPT at $\mathcal{O}(p^4)$ fixes the relevant counterterms in such a way that the $\pi^+\pi^- \to \pi^0\pi^0$ amplitude is still proportional to $(s - m_\pi^2)$ [20]. However, the advantage of using a framework where the scalar resonances are taken into account explicitly is that it allows to reproduce the scalar pole effects, a feature that is not possible in χPT.

As stated, the σ resonance formation effects should modify the previous results. In particular, instead of eq. (2.2) one now has to expect

$$\mathcal{A}(\pi^+\pi^- \to \pi^0\pi^0)_F = \frac{s - m_\pi^2}{f_\pi^2} F_\sigma(s)\,, \tag{2.4}$$

with an additional factor

$$F_\sigma(s) \equiv \frac{-m_\sigma^2 + k m_\pi^2}{D_\sigma(s)}\,, \tag{2.5}$$

accounting for σ exchange. A simple Breit-Wigner form, $D_\sigma(s) \equiv s - m_\sigma^2 + i m_\sigma \Gamma_\sigma$, where m_σ and Γ_σ are the effective mass and width, will be assumed for the σ propagator and contributions from $f_0(980)$ exchange will be neglected (they can be estimated to be below some 2 per thousand). Note that, as required, $F_\sigma(s) \to 1$ when $m_\sigma^2 \to \infty$ for any finite value of a free parameter k, thus recovering the chiral-loop result in eq. (2.2). It is important to remark that in eq. (2.4) we are not adding the σ contribution *ad hoc* but in a way that preserves the lowest order χPT amplitude once the σ resonance is decoupled. As mentioned before, the $\pi^+\pi^- \to \pi^0\pi^0$ amplitude is shown to be proportional to $(s - m_\pi^2)$ even when the σ formation effects are taken into account. This feature together with the recovery of the chiral result makes of the amplitude (2.4) a valid amplitude for studying such effects, thus making the whole analysis quite reliable.

We will consider two possible values of the parameter k: $k = 1$ and $k \simeq -2.5$. The first value corresponds to the Linear Sigma Model (LσM) [21]–[23] for scalar resonances, where the $\sigma\pi\pi$ coupling is given by $g_{\sigma\pi\pi} = (-m_\sigma^2 + m_\pi^2)/f_\pi$. Once inserted in

$$\Gamma_\sigma \simeq \Gamma(\sigma \to \pi\pi) = \frac{3}{32\pi} \frac{g_{\sigma\pi\pi}^2}{m_\sigma} \sqrt{1 - \frac{4m_\pi^2}{m_\sigma^2}}\,, \tag{2.6}$$

it predicts a σ (total) width around 300 MeV, which is only slightly below the value $\Gamma_\sigma \simeq$ 500 MeV favoured in refs. [2, 3]. This favoured value is reproduced if we enlarge the $g_{\sigma\pi\pi}$ coupling constant by fixing instead $k \simeq -2.5$. By itself, the amplitude for each one of these values of k ($k = 1$ in the LσM or $k \simeq -2.5$ in a more phenomenological context) inserted as the final factor in eq. (2.1) predicts, respectively,

$$\Gamma(\rho \to \pi^0\pi^0\gamma)_{\mathrm{L}\sigma\mathrm{M}} = 2.63\,\mathrm{keV}\,, \qquad \Gamma(\rho \to \pi^0\pi^0\gamma)_{\sigma\text{-phen}} = 1.84\,\mathrm{keV}\,. \tag{2.7}$$

The differences among the results in Eqs. (2.3) and (2.7) illustrate the effects of the $\sigma(500)$ resonance in $\rho^0 \to \pi^0\pi^0\gamma$ decays and seem to be large enough to establish both its existence and total width.

The same set of diagrams as before [10] contributes (apart from another set to be discussed later) to the amplitude for the charged channel $\rho^0 \to \pi^+\pi^-\gamma$. It similarly leads to

$$\mathcal{A}(\rho \to \pi^+\pi^-\gamma)_\chi = \frac{-eg}{\sqrt{2}\pi^2 m_{\pi^+}^2} \{a\}\, L(m_{\pi^+\pi^-}^2) \times \mathcal{A}(\pi^+\pi^- \to \pi^+\pi^-)_\chi\,, \tag{2.8}$$

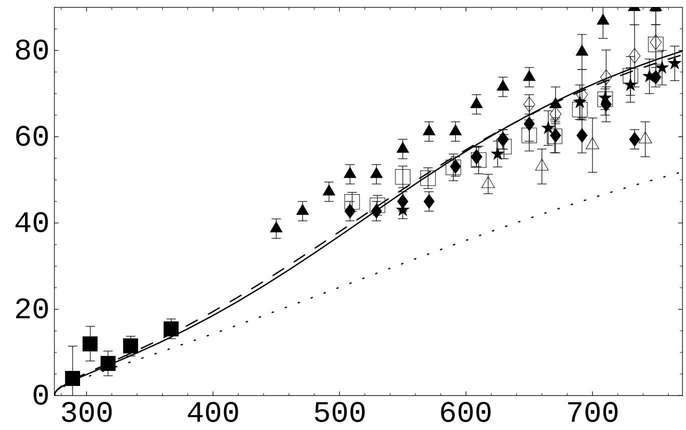

Figure 1: $\delta_0^0(s)$ (degrees) as a function of the dipion invariant mass $m_{\pi\pi}$ (MeV). The various predictions are for the σ models with $k=1$ (solid line), $k=-2.5$ (dashed line), and for the lowest order χPT (dotted line). Experimental data are taken from different analyses of the CERN-Munich Collaboration [26] (open and solid diamonds and triangles), as well as from [27] (stars), [28] (solid squares), and [29] (open squares).

where the four-pseudoscalar amplitude factorizes again in eq. (2.8) but now it is found to be proportional to the variable $s = m^2_{\pi^+\pi^-} = m_\rho^2 - 2m_\rho E_\gamma$:

$$\mathcal{A}(\pi^+\pi^- \to \pi^+\pi^-)_\chi = \frac{s}{2f_\pi^2}\,. \tag{2.9}$$

Integrating the photon energy spectrum over the whole physical region as before, one obtains

$$\Gamma(\rho \to \pi^+\pi^-\gamma)_\chi = 0.93\,\mathrm{keV}\,, \tag{2.10}$$

which is the simple chiral-loop prediction with no σ meson effects. These are easily introduced in terms of the previous $F_\sigma(s)$ factor accounting for σ resonance formation in the s-channel. Since isospin invariance forces this σ contribution to coincide with that for the previous neutral case, one unambiguously has

$$\mathcal{A}(\pi^+\pi^- \to \pi^+\pi^-)_F = \frac{s - m_\pi^2}{f_\pi^2} F_\sigma(s) - \frac{s/2 - m_\pi^2}{f_\pi^2}\,. \tag{2.11}$$

The final term is hard to interpret physically but it cannot be associated to σ formation in the s-channel and, as such, it does not contain the $F_\sigma(s)$ factor. It could be understood as the contribution of the exchange of all other intermediate resonances in the infinite mass limit. In any case, it is totally fixed by the need to recover the chiral-loop result (2.9) in the limit $m_\sigma^2 \to \infty$ or $F_\sigma(s) \to 1$. Once inserted as the final factor in eq. (2.8), the two terms in amplitude (2.11) lead to $\mathcal{A}(s)_\sigma + \mathcal{A}(\rho \to \pi^+\pi^-\gamma)_{\text{non-}\sigma}$, with a first k-dependent term accounting for σ exchange in the s-channel (as in the neutral case) and a second k-independent one. For $k=1$ (as in the LσM) or $k \simeq -2.5$ (as in the previous phenomenological context) one obtains

$$\Gamma(\rho \to \pi^+\pi^-\gamma)_{\mathrm{L}\sigma\mathrm{M}} = 5.21\,\mathrm{keV}\,, \qquad \Gamma(\rho \to \pi^+\pi^-\gamma)_{\sigma\text{-phen}} = 3.84\,\mathrm{keV}\,, \tag{2.12}$$

which, again, are markedly different from the value (2.10).

The predictions for $\rho \to \pi\pi\gamma$ in both channels are thus clearly different if one takes into consideration the effects of σ formation or not. One would expect that this difference to be also manifest in $\pi\pi$ scattering itself. In particular, one can calculate the effects of the σ resonance for the $I = J = 0$ $\pi\pi$ phaseshift $\delta_0^0(s)$ within the models with $k = 1$ (LσM) and $k = -2.5$ (phenomenological) and compare them with χPT at lowest order, i.e. with no σ resonance effects. Following ref. [24], $\delta_0^0(s) = \sqrt{1 - 4m_\pi^2/s}\, T_0^0(s)$ where T_0^0 is the partial wave with $I = J = 0$ obtained from the $\pi^+\pi^- \to \pi^0\pi^0$ amplitude (see also ref. [25]). A comparison of the different models with experimental data is shown in figure 1. Again, the models including σ meson effects offer a better description of data than the lowest order chiral prediction. In both σ models, with $k = 1$ and $k = -2.5$, a best fit to the data is achieved for the effective parameters $m_\sigma \simeq 500\,\mathrm{MeV}$ and $\Gamma_\sigma \simeq 500\,\mathrm{MeV}$, in agreement with refs. [2, 3].

3. $\rho^0 \to \pi^0\pi^0\gamma$

Apart from the previously discussed amplitude, the $\rho^0 \to \pi^0\pi^0\gamma$ decay is known to proceed also via ω-meson exchange in the t and u channels. Its evaluation offers no problems and has been performed by many authors with coincident results (see, for instance [30]). Explicitly, this background amplitude reads

$$\mathcal{A}(\rho \to \pi^0\pi^0\gamma)_\omega = \frac{G^2 e}{\sqrt{2} g}\left(\frac{P^2\{a\} + \{b(P)\}}{M_\omega^2 - P^2 - iM_\omega\Gamma_\omega} + \frac{P'^2\{a\} + \{b(P')\}}{M_\omega^2 - P'^2 - iM_\omega\Gamma_\omega}\right), \tag{3.1}$$

with $\{a\}$ the same as in eq. (2.8) and

$$\begin{aligned}\{b(P)\} \equiv\; & -(\epsilon^* \cdot \epsilon)\,(q^* \cdot P)\,(q \cdot P) - (\epsilon^* \cdot P)\,(\epsilon \cdot P)\,(q^* \cdot q) + \\ & + (\epsilon^* \cdot q)\,(\epsilon \cdot P)\,(q^* \cdot P) + (\epsilon \cdot q^*)\,(\epsilon^* \cdot P)\,(q \cdot P)\,,\end{aligned} \tag{3.2}$$

where $P = p + q$ and $P' = p' + q$ are the momenta of the intermediate ω meson in the t- and u-channel, respectively, and G is the strong and well known $\rho\omega\pi$ coupling constant [4, 13].

From this VMD amplitude alone and $G = \frac{3g^2}{4\pi^2 f_\pi}$ one easily obtains

$$\Gamma(\rho \to \pi^0\pi^0\gamma)_\omega = 1.89\,\mathrm{keV}\,, \tag{3.3}$$

in agreement with the results in refs. [4, 13], once the slight differences in numerical inputs are unified. The interference of this VMD (background) amplitude, $\mathcal{A}(\rho \to \pi^0\pi^0\gamma)_\omega$, with the (signal) amplitudes obtained before, $\mathcal{A}(\rho \to \pi^0\pi^0\gamma)_\chi$, $\mathcal{A}(\rho \to \pi^0\pi^0\gamma)_{\mathrm{L}\sigma\mathrm{M}}$ or $\mathcal{A}(\rho \to \pi^0\pi^0\gamma)_{\sigma\text{-phen}}$, is found to be constructive in the whole kinematical region in the three cases and one globally has

$$\begin{aligned}\Gamma^{\chi+\omega}_{\rho\to\pi^0\pi^0\gamma} &= 4.40\,\mathrm{keV}\,, & B^{\chi+\omega}_{\rho\to\pi^0\pi^0\gamma} &= 2.95 \times 10^{-5}\,,\\ \Gamma^{\mathrm{L}\sigma\mathrm{M}+\omega}_{\rho\to\pi^0\pi^0\gamma} &= 6.29\,\mathrm{keV}\,, & B^{\mathrm{L}\sigma\mathrm{M}+\omega}_{\rho\to\pi^0\pi^0\gamma} &= 4.21 \times 10^{-5}\,,\\ \Gamma^{\sigma\text{-phen}\,+\omega}_{\rho\to\pi^0\pi^0\gamma} &= 5.10\,\mathrm{keV}\,, & B^{\sigma\text{-phen}\,+\omega}_{\rho\to\pi^0\pi^0\gamma} &= 3.42 \times 10^{-5}\,.\end{aligned} \tag{3.4}$$

The corresponding spectra have been plotted in figure 2 from which the effects of the σ formation and their dependence on the σ width can be observed. The fact that our various

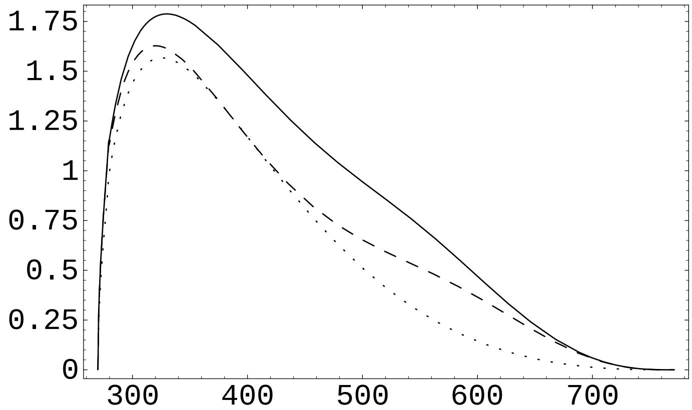

Figure 2: $dB(\rho \to \pi^0\pi^0\gamma)/dm_{\pi^0\pi^0} \times 10^7$ (MeV^{-1}) as a function of the dipion invariant mass $m_{\pi^0\pi^0}$ (MeV). The various predictions are for the input values: $m_\sigma = 500\,\mathrm{MeV}$ and $\Gamma_\sigma = 300\,\mathrm{MeV}$ (solid line); $m_\sigma = 500\,\mathrm{MeV}$ and $\Gamma_\sigma = 500\,\mathrm{MeV}$ (dashed line). The chiral-loop prediction with no scalar pole is also included for comparison (dotted line).

signal amplitudes are important compared with the background contribution and that their interferences are positive makes this $\rho \to \pi^0\pi^0\gamma$ decay mostly appropriate to reveal the σ meson effects [23].

On the experimental side, the SND Collaboration has reported very recently a new measurement of the $\rho^0 \to \pi^0\pi^0\gamma$ decay. For the branching ratio, they obtain [4]

$$B(\rho \to \pi^0\pi^0\gamma) = (4.1^{+1.0}_{-0.9} \pm 0.3) \times 10^{-5}\,, \tag{3.5}$$

and therefore $\Gamma(\rho \to \pi^0\pi^0\gamma) = (6.1^{+1.6}_{-1.4})$ keV. This new value is in agreement with the first measurement [5]

$$B(\rho \to \pi^0\pi^0\gamma) = (4.8^{+3.4}_{-1.8} \pm 0.2) \times 10^{-5}\,. \tag{3.6}$$

Comparison with our predictions indicates that a substantial σ meson contribution is needed. Unfortunately, more crucial data on the $\pi^0\pi^0$ invariant mass spectrum have not been reported yet.

4. $\rho^0 \to \pi^+\pi^-\gamma$

The background amplitude for this charged decay mode is more involved than for the previous, neutral case. Apart from the additional amplitude, $\mathcal{A}(\rho \to \pi^+\pi^-\gamma)_{\text{non-}\sigma}$, generated by the second term in eq. (2.11), i.e. the term not linked to σ formation in the s-channel, we have further t- and u-channel contributions. The dominant one, particularly for low photon energies, is the bremsstrahlung amplitude, $\mathcal{A}(\rho \to \pi^+\pi^-\gamma)_{\text{brems}}$, while the other one originates from $a_1(1260)$ contributions. They can be regarded as $J^{PC} = 0^{-+}$ and 1^{++} exchanges in the t and u channels. Both contributions can be related to tree-level amplitudes and to the set of one-loop diagrams which are specific for $\gamma\gamma \to \pi^+\pi^-$ in the analysis of ref. [10]. Contrasting with the previously discussed chiral-loop diagrams

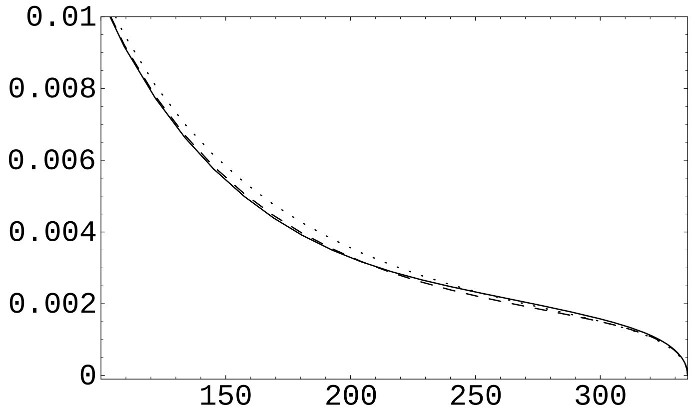

Figure 3: $d\Gamma(\rho \to \pi^+\pi^-\gamma)/dE_\gamma$ as a function of the photon energy E_γ (MeV) for $E_\gamma > 100\,\mathrm{MeV}$. The various predictions are for the input values: $m_\sigma = 500\,\mathrm{MeV}$ and $\Gamma_\sigma = 300\,\mathrm{MeV}$ (solid line); $m_\sigma = 500\,\mathrm{MeV}$ and $\Gamma_\sigma = 500\,\mathrm{MeV}$ (dashed line). The chiral-loop prediction with no scalars is also included for comparison (dotted line).

— which contributed to both the neutral and charged decay channels with finite corrections — this new set includes divergent vertex corrections and mass insertions. One of these divergences, appearing only in our case with $(q^*)^2 = m_\rho^2 \neq 0$, requires the contribution of a ρ-dominated counterterm which leads to an amplitude, $\mathcal{A}(\rho \to \pi^+\pi^-\gamma)_{\mathrm{brems}}$, including the one-loop effects in the physical value of the $\rho\pi\pi$ coupling constant g (those for the real photon vanish with $q^2 = 0$). The other piece requires the term in ref. [10] containing the combination of low-energy constants $L_9^r + L_{10}^r$ which is known to be saturated by pure axial resonance exchange [11]. It thus generates an $a_1(1260)$ contribution, $\mathcal{A}(\rho \to \pi^+\pi^-\gamma)_{a_1} = 16\sqrt{2}ge(L_9^r + L_{10}^r)\{a\}$, which for $L_9^r + L_{10}^r \simeq 1.4 \times 10^{-3}$ is well below our signal amplitude, and can safely be neglected.

The remaining, bremsstrahlung contribution is well known [30]–[32]

$$
\begin{aligned}
\mathcal{A}(\rho \to \pi^+\pi^-\gamma)_{\mathrm{brems}} = 2\sqrt{2}eg\Bigg[\epsilon^* \cdot \epsilon - \frac{1}{2}\left(\frac{\epsilon \cdot p_+}{q \cdot p_+} + \frac{\epsilon \cdot p_-}{q \cdot p_-}\right)\epsilon^* \cdot q - \\
- \frac{1}{2}\left(\frac{\epsilon \cdot p_+}{q \cdot p_+} - \frac{\epsilon \cdot p_-}{q \cdot p_-}\right)\epsilon^* \cdot (p_+ - p_-)\Bigg] ,
\end{aligned}
\tag{4.1}
$$

and, by itself, it leads to

$$
\Gamma(\rho \to \pi^+\pi^-\gamma)_{\mathrm{brems}} = 1.706\,\mathrm{MeV} \quad \text{for } E_\gamma > 50\,\mathrm{MeV} \ . \tag{4.2}
$$

The various signal amplitudes, $\mathcal{A}_\sigma(s)$, have to be added to that from background, $\mathcal{A}_{\mathrm{backg}} \equiv \mathcal{A}_{\mathrm{brems}} + \mathcal{A}_{\text{non-}\sigma} + \mathcal{A}_{a_1} \simeq \mathcal{A}_{\mathrm{brems}} + \mathcal{A}_{\text{non-}\sigma}$. This leads to the predictions displayed in figure 3 for $E_\gamma > 100\,\mathrm{MeV}$, where σ meson exchange effects are visible. These are moderately dependent on the σ width but show a depletion of events below $E_\gamma \simeq 250\,\mathrm{MeV}$ when compared to the chiral amplitude. The integrated results for $E_\gamma > 50\,\mathrm{MeV}$, being dominated by bremsstrahlung at low E_γ, are less interesting but included for completeness

$$\begin{aligned}
\Gamma^{\chi+\text{backg}}_{\rho\to\pi^+\pi^-\gamma} &= 1.748\,\text{MeV}\,, & B^{\chi+\text{backg}}_{\rho\to\pi^+\pi^-\gamma} &= 1.171\times 10^{-2}\,,\\
\Gamma^{\text{L}\sigma\text{M}+\text{backg}}_{\rho\to\pi^+\pi^-\gamma} &= 1.698\,\text{MeV}\,, & B^{\text{L}\sigma\text{M}+\text{backg}}_{\rho\to\pi^+\pi^-\gamma} &= 1.138\times 10^{-2}\,,\\
\Gamma^{\sigma\text{-phen}+\text{backg}}_{\rho\to\pi^+\pi^-\gamma} &= 1.696\,\text{MeV}\,, & B^{\sigma\text{-phen}+\text{backg}}_{\rho\to\pi^+\pi^-\gamma} &= 1.136\times 10^{-2}\,.
\end{aligned} \tag{4.3}$$

For the $\rho^0 \to \pi^+\pi^-\gamma$ decay, the present experimental branching ratio is [6, 7]

$$B(\rho\to\pi^+\pi^+\gamma) = (0.99\pm 0.04\pm 0.15)\% \qquad \text{for } E_\gamma > 50\,\text{MeV}\,, \tag{4.4}$$

quite compatible with all our results. The observed photon spectrum compares rather favorably with pure bremsstrahlung emission except (possibly) for the last bin, where the σ amplitudes moderately contribute to improve the agreement. Finally, a model-independent upper limit of the branching ratio of the $\rho^0 \to \pi^+\pi^-\gamma$ decay via scalar resonance exchange was found to be $B(\rho\to\pi^+\pi^+\gamma) < 5\times 10^{-3}$ (90% CL) [6, 7] and thus fully respected in our approach.

5. Comments and conclusions

Apart from old attempts to identify σ meson contributions to $\rho^0 \to \pi\pi\gamma$ decays [33], other authors have reconsidered the issue more recently. Oset and collaborators [34, 35], for instance, have discussed these processes in their unitarized chiral-loop approach where the σ meson pole is dynamically generated; this makes their approach, as well as their results, quite different from ours. The same happens with another series of papers by Gokalp *et al.* [36]–[38], where σ meson effects are *added* to the chiral-loop contribution; in this way, an attractive feature of our treatment, namely, that in the limit of high m_σ one recovers the expected and well defined chiral-loop amplitude, is lost.

In conclusion, $\rho^0 \to \pi\pi\gamma$ decays have been shown to be an important source of information on the low-mass $\pi\pi$ spectrum in the s-channel. A global analysis of both processes, with a common amplitude interfering with markedly different but well stablished backgrounds, should contribute to clarify the σ meson status. According to our analysis, present data already suggest the existence of such a low-mass state. Moderately improved data on $\rho^0 \to \pi\pi\gamma$ decays could be decisive to settle the issue.

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